or if $\frac{n+m+p}{2}=s$.

$$\int_{-1}^{1} P_m P_n P_p d\mu = \frac{2}{2s+1} \frac{A(s-m)A(s-n)A(s-p)}{A(s)}$$

where as above

$$A(m)=\frac{1\,.\,3\,.\,5\,.\,.\,.\,.\,(2m-1)}{1\,.\,2\,.\,3\,.\,.\,.\,.\,m}=2^m\,.\,\frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdots\cdot\left(m-\frac{1}{2}\right)}{1\,.\,2\,.\,3\,.\,.\,.\,m}$$

It is clear that, in order that this integral may be finite, no one of the quantities m, n, and p must be greater than the sum of the other two, and that $m+n+p$ must be an even integer.

I learn from Mr. Ferrers that, in the course of the year 1874, he likewise obtained the expression for the product of two Legendre's coefficients, by a method very similar to mine. In his work on "Spherical Harmonics," recently published, he gives, without proof, the above result for the value of the definite integral $\int_{-1}^{1} P_m P_n P_p d\mu$.

IV. "Experiments on the Colours shown by thin liquid Films under the Action of Sonorous Vibrations." By SEDLEY TAYLOR, M.A., late Fellow of Trinity College, Cambridge. Communicated by J. W. L. GLAISHER, M.A., F.R.S. Received December 12, 1877.

(Plates 5 and 6.)

Professor Helmholtz remarks, at page 603 of the fourth edition of his "Tonempfindungen," that a film of soapsuds and glycerine forms, when caused to occupy the orifice of one of his "resonators," an extremely sensitive means by which to make visible the vibrations of the air within its cavity.

While I was engaged in verifying this observation, my notice was attracted to the parallel bands of colour which traversed the film, and it occurred to me to examine whether the forms of these bands were affected by the sonorous vibrations which agitated the film. A few rough trials having convinced me that they were so affected, I at once proceeded to submit the phenomena which presented themselves to a closer examination.

Having caused a film to adhere to the circular aperture of a Helmholtz resonator, and allowed the fluid to drain off until the interference-colours became visible, I set the resonator, nipple downwards, in a stand, so that the film was exactly horizontal, and then stroked with a resined bow a tuning-fork of the same pitch mounted on its resonance

box and placed near the resonator. With a tenacious film and uniform bowing, this resulted in the formation of a fixed figure, consisting of coloured bands, straight or curved, symmetrically arranged, and generally accompanied by one or more colour-vortices rotating in opposite directions. A single sweep of the bow usually sufficed to call out such a figure from a previous state of colour-chaos, and, when it was once established, careful bowing would keep it fairly steady for some little time. While, however, its form remained thus constant, its colours underwent a progressive series of changes as the thickness of the film gradually diminished. Presently there would come a complete break-up of the figure, often accompanied by a violent rotation of the whole film, after which a different figure, generally of a simpler form, would establish itself, to be in its turn, provided the film only held out long euough, replaced by another and still more simple figure. Sometimes, but much less frequently, the changes of form occurred without a separating interval of disorder, the incoming figure supplanting its predecessor with kaleidoscopic abruptness. The most interesting feature of these figures was their stationary colour-vortices, arranged pairwise, and churning round and round with a velocity which seemed limited only by the loudness of the sound at command, and by the capacity of the film for bearing the strain put upon it. The fixed bands, viewed at any one instant, mostly presented a surprising degree of complexity, with entire symmetry in the arrangement both of form and of vividly contrasting colour. The general effect may, perhaps, be compared to that of an elaborate "set-piece" of fireworks, in which turbines of coloured flame play a conspicuous part. But the great charm of the experiment lay in watching the successive changes of form and hue which, beautiful from the very outset, became, as the film neared its moment of dissolution, surpassingly gorgeous. It would, I think, be difficult to point to a more splendid series of phenomena in the whole range of physical optics.

To give anything like a complete representation of the colour-appearances observed would severely tax the utmost resources of pictorial art. The illustrations* appended to this paper are meant only to convey a general notion of the phenomena which presented themselves. The memoranda for them were carefully taken down during actual observation of the experiments, so that, in their broader features, the figures given may be safely depended on. Minute accuracy of detail could not, in dealing with such wayward phenomena, be secured. except by a greater expenditure of time than was feasible. I propose to make a few remarks on some of the colour-figures produced by the mode of experimenting described above.

Gentle bowing, and a film not too thin, usually called out fig. 1, in

* Drawn by Mr. Daniel Wood, Master of the Cambridge School of Art, who most kindly placed his skill and scanty leisure at my disposal.

which bright concentric rings contrasted with a differently tinted ground, while the whole was surrounded by a margin of mixed colours in an unorganised condition. Stronger bowing, or the progressive thinning of the film, would then establish fig. 2, the most typical and persistent of the whole series. Here concentric circular bands were enclosed by others approximately square, and these again, together with four sets of irregularly flattened ovals symmetrically disposed, by other square bands, inclined at 45° to the former ones. Outside the latter were four pairs of colour-vortices, with stationary tadpole-shaped nuclei, rotating in the alternate directions shown by the arrows. Nearly the whole space included between the outer square and the edge of the film was occupied by colours whirling in circular or slightly elliptic orbits. The contrast between the fixed and moving portions of this figure was always extremely striking and beautiful.

In fig. 3, a set of fixed concentric rings separated adjacent pairs of vortices.

Figs. 4 and 5 presented a pentagonal arrangement with five pairs of vortices. The dotted space in the latter figure appeared speckled over with minute air-bubbles.

Fig. 6 belonged to a less regular class of forms.

Fig. 7 was only once observed. Its central rings were inclosed in a series of triangular bands, outside which were three sets of flattened ovals similar to those in fig. 2, but each containing a single vortex rotating about a nucleus.

Fig. 8 usually showed itself only when the thinning of the film was far advanced. There was a flow of colour along the dotted lines in the direction of the arrow, which subsequently divided into two streams, and, after passing outside the two sets of flattened ovals and through the channels separating them from a third irregular crescent-shaped series of bands, united again opposite the protuberance in the concavity of the latter, and went on performing the same circuit. When the film had become excessively thin, this figure frequently showed nothing but the two tadpole nuclei, with an oval vortex about each, whose longer diameter was not much less than that of the film itself.

In all the preceding figures the axis of symmetry was evidently determined by the direction in which the sonorous vibrations reached the film.

The results which have been as yet described were obtained from films clinging to the circular orifice of a resonator. A simpler mode of proceeding is to form the film on an aperture cut in a piece of card-board or thin sheet of metal, and place this upon the open end of a resonance-box, into which its appropriate tuning-fork has been previously screwed. The box is, of course, to be held steady, with its opening horizontal, while the fork is thrown into vibration. By

operating in this manner with apertures of various shapes, I obtained a large number of distinct figures: a few of the least complex have been selected for illustration.

In figs. 9, 10, 11, the aperture used was an equilateral triangle; in figs. 12, 13, 14, a square. The general character of these results will, after the explanations already given, be readily understood on inspection of the diagrams. Their axis of symmetry depended on the position of the film with reference to that of the tuning-fork.

When larger apertures, or a fork and box of higher pitch, were used, the resulting figures became rapidly more complex, and forms were readily obtained in which the whole film was covered with an uniform pattern consisting of some single figure analogous to one of those already described repeated over and over again. With very acute sounds the separate figures became too small for recognition and too numerous for counting.

The resonance of an air-cavity is, as is well known, not limited to supporting a note of one single degree of pitch, but can also reinforce other notes, provided they are not too far distant from its proper or fundamental tone. It was therefore to be expected that colour-figures could be obtained from a film in the mouth of a resonator by employing sounds other than that to which its cavity was tuned. In putting this to the test of experiment I found it convenient to replace the sounds of mounted tuning-forks by those of my own voice. The film, formed as at first in the orifice of a resonator, showed itself sensitive to very slight differences of pitch. When the note sung was somewhere in the neighbourhood of the resonator's proper tone, the smallest sharpening or flattening which my voice could produce led to an instantaneous and unmistakable change of figure. Moreover, the limits of pitch on either side of the resonator's own note, within which permanent figures could be obtained, were considerably wider than those between which audible reinforcement by the resonator itself occurred.

In order to test the capacity of two figures corresponding to tones of different pitch for coalescing into a regular resultant-form, I placed a film symmetrically with respect to the open ends of two resonance-boxes belonging to forks forming consonant intervals with each other. By exciting either fork separately a steady figure was obtained, but the coexistence of their sounds caused an immediate tumultuous movement from which no permanent joint form emerged. There seemed to be an absolute incompatibility between the two figures which made all compromise impossible. Desirous of ascertaining whether this repugnance extended to the case of sounds all but identical in pitch, I used a pair of unison forks, one of which had been slightly flattened so as to beat about twice per second with its fellow. Some very interesting phenomena were the result. The fixed

portions of the approximately identical figures due to either fork by itself, took up, while both were sounding, a swaying movement about their mean position; one complete oscillation of figure synchronizing exactly with each beat* heard. The resonance-boxes had, in this experiment, been placed with their openings exactly opposite each other, and the film, which was rectangular in shape, midway between, but in a horizontal plane slightly above them. The swaying motion was, under these circumstances, on the whole rectilinear, as though each fork alternately gave the entire figure a pull in its own direction. The behaviour of the vortices was still more remarkable. With vigorous and equal bowing they rotated several times in one direction during the first half of each beat, and the same number of times in the opposite direction during the second half of it. If, instead of occupying the relative positions above described, in which the forks when sounding singly gave rise to antagonistic vortices, they were both placed on one side of the film, the result was rotation during one half-beat and inaction during the next, followed again by similar alternations, but the direction of rotation remaining constant throughout. In this case the vortices moved most rapidly during the maximum and rested during the minimum of intensity. But in others it was not so, and I even observed instances where in one part of the figure the rotation coincided with the maximum and the quiescence with the minimum, while at another part of it the exactly contrary state of things prevailed.

In every experiment hitherto described the film was either acted on by the resonance of a spherical air-cavity, which practically reinforces only the fundamental tone of a compound sound in unison with it; or else the vibrations employed were exclusively those of a mounted tuning-fork which follow the pendulum law. Hence no other kind of movement was transmitted save that which gives rise to what Helmholtz calls a "simple tone." In order to examine the effects produced by composite sounds, it was desirable to let their vibrations act on a film unconnected with any resonant cavity. For this purpose one end of a caoutchouc tube of large bore was fitted into a metal ring fixed in a horizontal plane on which the film-bearing discs could be placed. Notes of the human voice, of tuning forks, organ pipes, &c., being sounded into the tube, either directly or through a funnel in the shape of an ordinary ear-trumpet, their effects on films of various forms and sizes could be conveniently observed. A very wide field for research was thus opened up, which I do not propose to enter upon here beyond simply mentioning one result obtained in this manner which possesses an independent interest. When two notes, identical in pitch and

* The absence of an English equivalent for the German *Schwebung*, which denotes the whole phenomena from one maximum of intensity (*Schlag*) to the next, is most inconvenient, and makes itself very perceptible here.

loudness, but differing markedly in what is called quality or *timbre*, were alternately sounded, two perfectly distinct figures were obtained, each presenting itself again and again for many alternations the instant its own note was sounded, and remaining constant until that note ceased. By this apparatus, therefore, permanent pictures of the relative quality of musical sounds may be secured.

I content myself here with a description of the phenomena I have observed, and make no attempt at determining the mechanical conditions under which they occur. It may, however, be worth while to remark that the most striking feature of the figures above described, the vortices, can be exactly reproduced with the caoutchouc tube apparatus by gently sucking a little air through it, taking care that the whole opening of the tube is not in simultaneous contact with the lips, and that its other end is not completely closed by the disc and film.

Before concluding I wish to draw attention to some allied phenomena described by Mr. E. B. Tylor in "Nature," for May, 1877, p. 12. Distinct patterns were obtained by him, but it would appear from the directions he gives for producing "a film more free from interference-colours, so as to display the vibration-figures on an almost clear ground" that no permanent colour-patterns are in question in his letter. He speaks indeed of "the gorgeous scenic effect of the masses of prismatic colour whirled hither and thither by the musical vibrations," but of nothing more fixed and regular. The mode of experimenting adopted by him on that occasion suggested the third form of apparatus described in the present paper.

February 7, 1878.

Sir JOSEPH HOOKER, K.C.S.I., President, in the Chair.

The Presents received were laid on the table and thanks ordered for them.

The following Papers were read:—

I. "On the Comparison of the Standard Barometers of the Royal Observatory, Greenwich, and the Kew Observatory." By G. M. WHIPPLE, B.Sc., Superintendent of the Kew Observatory. Communicated by order of the Kew Committee, WARREN DE LA RUE, F.R.S., Vice-Chairman. Received November 26, 1877.

The Standard Barometers of these two important establishments, up to within a recent date, had never been compared directly, although